

Several Examples on Solving an ODE using MATLAB

Introduction:

A Tutorial on Solving an ODE through the use of the discretization method then using the MATLAB ODE solver and comparing the results. Two examples are presented the first with detailed solution while the second only the MATLAB part is provided .

Example 1:

The ODE equation to solve is the following one

$$y' = 2xy + x$$

The numerical solution comes through:

$$y' = 2x(i)y(i) + x(i)$$

We can represent the y derivative in the following form

$$y' = \frac{dy}{dx} = \frac{y(i+1) - y(i)}{dx}$$

We then substitute the formula derivative into the ode equation:

$$\frac{y(i+1) - y(i)}{dx} = 2x(i)y(i) + x(i)$$

The resulting ode should look something like this:

$$y(i+1) = (2x(i)y(i) + x(i))dx + y(i)$$

The length of the studied region on the x axis is referred to as L:

$$L = L0 + x0$$

x0 is the initial value for the studied range, while L0 is the length of the added to x0.

The discrete length depends on the studied length and number of points for the studied length.

$$dx = \frac{L}{N}$$

To solve the equation there is a need to provide the initial value for the $y(1) = 0.1$ and $x(1) = 1$. Also there is a need to provide the analysis range on the x axis

The Linear first order equation is solved by:

$$y' + p(x)y = q(x) \quad \text{Comparing} \quad y' = 2xy + x$$

We find that:

$$p(x) = -2x \quad \text{and} \quad q(x) = x$$

Calculating the integrating factor:

$$I(x) = e^{\int p(x)dx} = e^{\int -2x dx}$$

The integrating factor is:

$$I(x) = e^{(-x^2)}$$

Multiplying the equation by the integration factor

$$\frac{d(yI)}{dx} = Iq(x)$$

$$y'e^{(-x^2)} - 2xye^{(-x^2)} = xe^{(-x^2)}$$

$$\frac{d(ye^{(-x^2)})}{dx} = xe^{(-x^2)}$$

Integrating Both sides:

$$\int \frac{d(ye^{(-x^2)})}{dx} dx = \int \frac{-2}{-2} xe^{(-x^2)} dx$$

The initial form of the solution is:

$$ye^{(-x^2)} = -0.5e^{(-x^2)} + c$$

We have $x_0 = 1$ and $y_0 = 0$ for our studied case, where $c = 0.5e^{(-x_0^2)} + y_0e^{(-x_0^2)}$.

Therefore $c = 0.5e^{-1}$.

Now we conduct a couple of cosmetic operations:

$$y = ce^{x^2} - 0.5$$

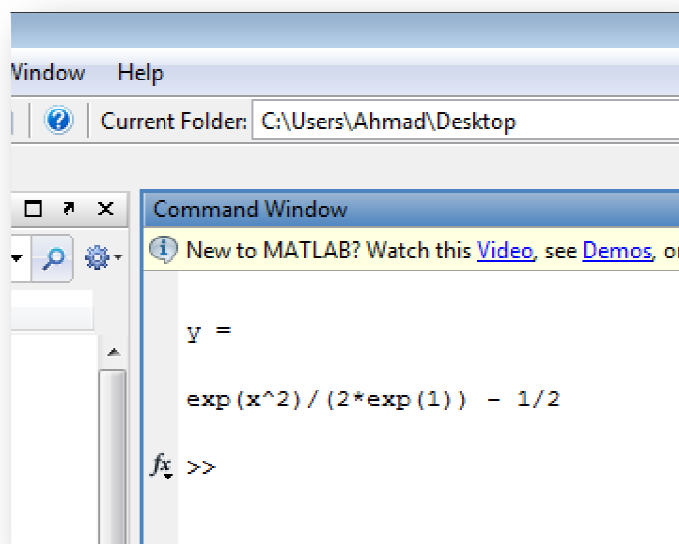
The final form of the solution:

$$y = 0.5e^{-1}e^{x^2} - 0.5$$

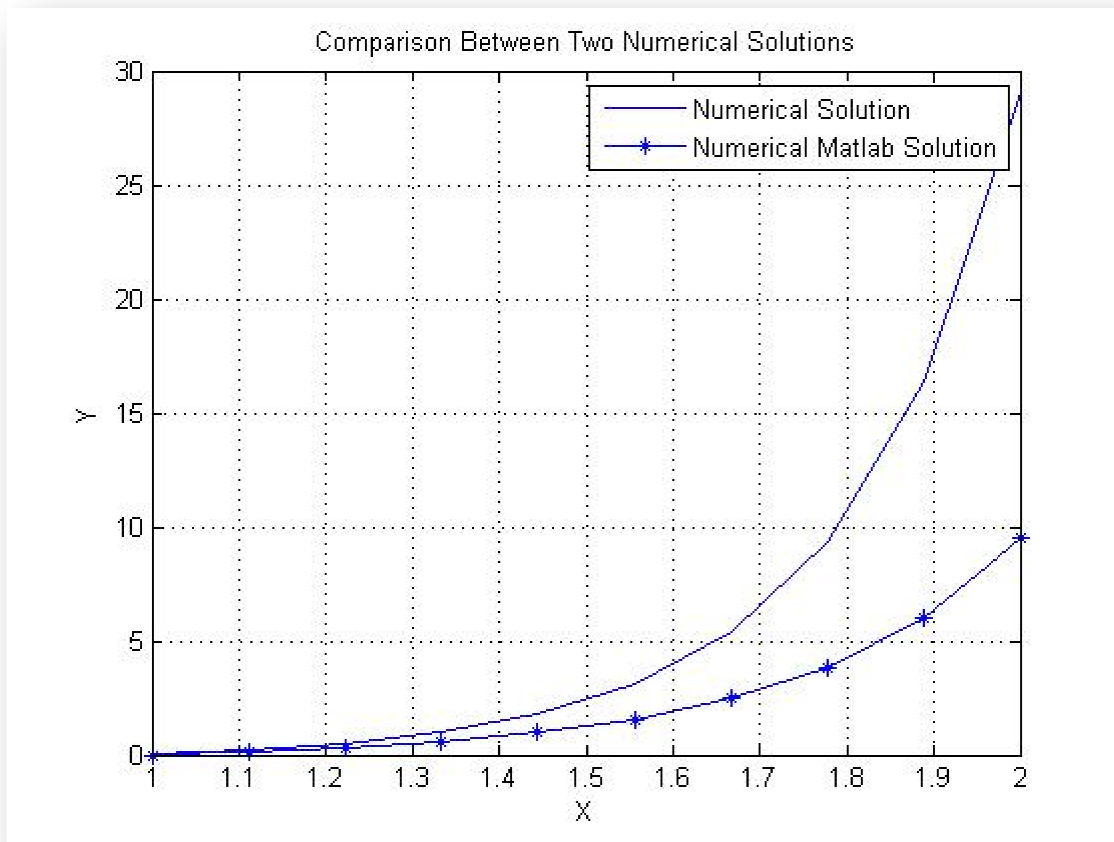
The MATLAB code is:

```
clc
clear
N=10;
x0=1;
L0=1;
L=L0+x0;
dx=L/N;
y(1)=0;
x(1)=x0;
x=linspace(x0,L,N);
for i=1:N-1;
y(i+1)=dx*(2*x(i)*y(i)+x(i))+y(i);
end
plot(x(:),y(:))
grid on
hold on
eqn1 = 'Dy = 2*x*y+x';
y=dsolve(eqn1,'y(1)=0','x')
x=linspace(x0,L,N);
z=eval(vectorize(y));
plot(x,z,'-*')
title('Comparison Between Two Numerical Solutions')
xlabel('X')
ylabel('Y')
legend('Numerical Solution','Numerical Matlab Solution')
```

The MATLAB code should give the following solution for the ODE:



The output of the provided code should look like this:



Example 2:

The ODE equation to solve is the following one

$$y' = 5x + 3$$

The numerical solution comes through:

$$y' = 5x(i) + 3$$

We can represent the y derivative in the following form

$$y' = \frac{dy}{dx} = \frac{y(i+1) - y(i)}{dx}$$

We then substitute the formula derivative into the ode equation:

$$\frac{y(i+1) - y(i)}{dx} = 5x(i) + 3$$

The resulting ode should look something like this:

$$y(i+1) = (5x(i) + 3)dx + y(i)$$

The length of the studied region on the x axis is referred to as L:

$$L = L0 + x0$$

x0 is the initial value for the studied range, while L0 is the length of the added to x0.

The discrete length depends on the studied length and number of points for the studied length.

$$dx = \frac{L}{N}$$

To solve the equation there is a need to provide the initial value for the $y(1) = 0.1$ and $x(1) = 1$. Also there is a need to provide the analysis range on the x axis

The MATLAB code is:

```

clc
clear
N=20;
x0=1;
L0=10;
L=L0+x0;
dx=L/N;
y(1)=0;
x(1)=x0;
x=linspace(x0,L,N);
for i=1:N-1;
y(i+1)=dx*(5*x(i)+3)+y(i);
end
plot(x(:),y(:))
grid on
hold on
eqn1 = 'Dy = 5*x+3';
y=dsolve(eqn1,'y(1)=0','x')
x=linspace(x0,L,N);
z=eval(vectorize(y));
plot(x,z,'-*')
title('Comparison Between Two Numerical Solutions')
xlabel('X')
ylabel('Y')
legend('Numerical Solution','Numerical Matlab Solution')

```

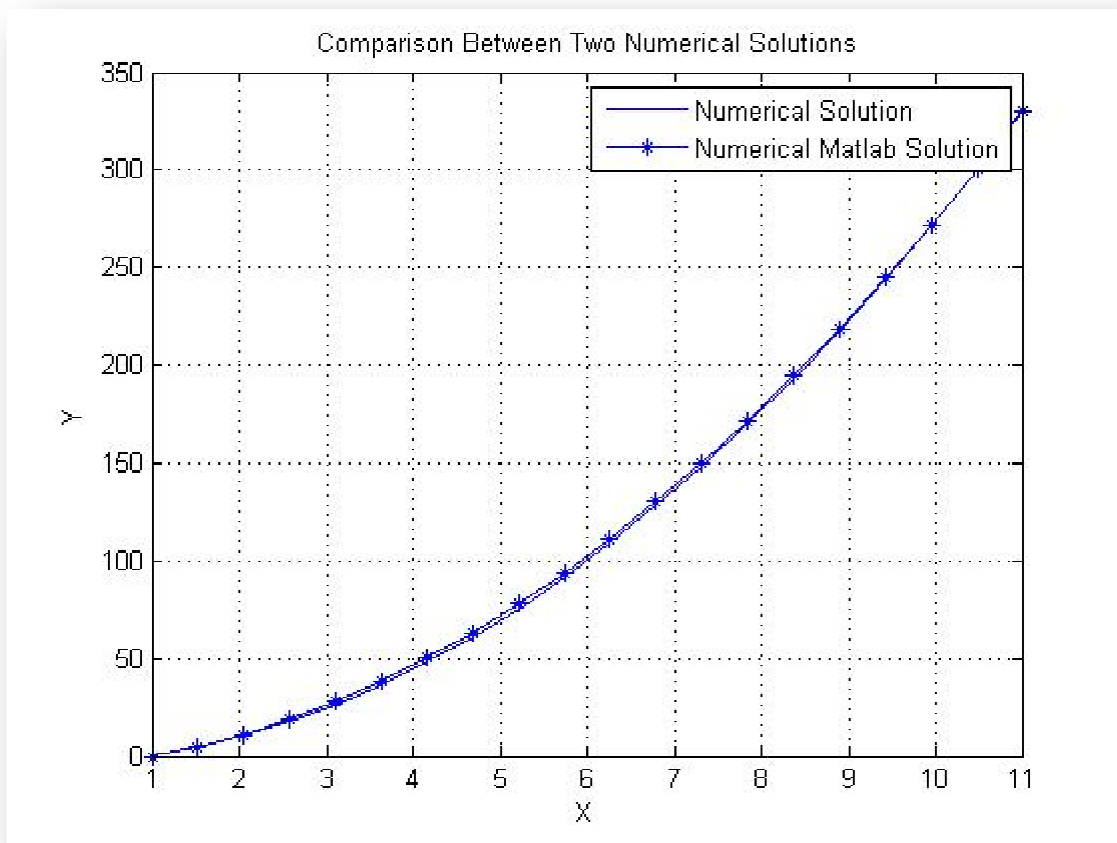
The MATLAB code should give the following solution for the ODE:

```
Help
Current Folder: C:\Users\Ahmad\Desktop

Command Window
New to MATLAB? Watch this Video, see

y =
(5*x + 3)^2/10 - 32/5
fx >> |
```

The output of the provided code should look like this:



That's the end of the tutorial I hope that was helpful please feel free to contact me regarding any mistakes or recommendations for the tutorial.

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